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FINAL REPORT

# SYSTEMS THEORY APPLICABLE TO AIR FORCE COMMUNICATION AND INFORMATION HANDLING PROBLEMS

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## ABSTRACT

This report summarizes the results of five years of research in the general areas of algebraic systems and stochastic systems. Topics considered related to algebraic systems include: unequal error protection codes, burst and multiple burst correcting codes, codes designed for the Lee metric, improved BCH codes, decoding of BCH codes, system reliability through algebraic coding, channel modeling of real channels, the use of a time dispersive channel as an encoder and the study of certain algebraic sequences. In the study of stochastic systems the following topics were considered: optimum sampling, invariant estimators, feedback communications, adaptive detection and learning, high speed binary data transmission over a-band-limited channel, multiple access communications and rate distortion functions for stochastic sources.

Only a brief summary of the research results is presented. However, abstracts of papers and dissertations resulting from this research are included.

# I. INTRODUCTION

This is the final report on Contract AF49(638)-1600 undertaken with the Air Force Office of Scientific Research, United States Air Force. This contract was in effect from September 16, 1965 to September 15, 1970. Various research topics were studied in the general area of communications theory and information theory. This research was also concerned with the relationship of these areas to automata and control theory.

In this section, a brief summary of the research completed is presented; abstracts of published papers and dissertations which are concerned with these research topics are given in later sections. Detailed discussions of this research are contained in the actual published papers and dissertations.

The research areas investigated fall into two categories which will be termed algebraic systems and stochastic systems. The studies in algebraic systems include several topics in algebraic coding as well as studies of algebraic sequences with desirable properties. The algebraic codes studied have applications in the reliable transmission of digital data over noisy channels, the reliable storage and recovery of data from high density storage media and data compression. The algebraic sequences considered have applications in wide band communications, multiple access communications, multiplexing, addressing, pattern recognition and encipherment.

The studies in stochastic systems include several topics in signal detection and estimation, signal design, feedback communications, channel modeling and learning machines.

Algebraic Systems

Various methods of algebraic coding were considered in this investigation. These include unequal error protection codes, burst and multiple burst correcting codes, codes based upon interleaved algebraic sequences, codes designed for the Lee metric, improved BCH codes, methods of decoding BCH codes and methods of improving the reliability of digital systems by coding.

Unequal error protection codes have the capability of providing different levels of error protection for specific digits of a code word. Such codes may have application in PCM transmission of quantized sampled data where errors in certain digits would be more costly in reconstruction than errors in other digits. Methods of synthesizing such codes and decoding algorithms have been found. Upper and lower bounds on the redundancy have also been derived. (See II-2 and IV-8)

Several classes of single burst and multiple burst correcting algebraic codes have been studied. The concept of a burst distance was defined and the burst correcting capabilities of codes was related to the burst distance of the codes. The multiple burst correcting capability of product codes was investigated. This class of codes is particularly interesting because of an easy decoding algorithm developed for them. (See III-6 and IV-17.)

A new class of nonlinear random error correcting codes based upon the interleaving of periodic algebraic sequences was developed. The performance of such codes is roughly the same as the best, presently known, codes, the BCH codes. (See IV-4.)

A study was made of codes designed for the Lee (rather than the Hamming) metric. Such codes would appear to have application in a M-phase or M-amplitude communications system where M is a prime greater than 2. General properties of such codes have been derived. Also specific codes have been synthesized for correcting random errors or burst errors. (See III-9.)

An interesting duality was discovered between a decoding algorithm for BCH codes and a common method of curve fitting. (See IV-7.) Also it was found that two information symbols can be appended to certain BCH codes without effecting the guaranteed minimum distance of these codes. The lengthened codes are then used as building blocks for perfect single-error correcting codes in another metric. (See IV-14.)

The techniques of algebraic coding theory were used to improve the reliability of digital devices. Redundant circuits were designed and evaluated based upon different classes of algebraic codes. (See IV-15.)

Finally, two investigations were concerned with the use of codes over real communication channels. The first of these was concerned with developing a model for the errors which occur on a real channel. A model was developed which could match the error distributions recorded from different types of channels, for which the parameters of the model could be easily determined and for which the performance of codes could be evaluated quite simply. (See III-3 and IV-10.) The second investigation was concerned with the similarities between a time dispersive channel and an encoder. In this investigation, a time dispersive channel was equalized so as to act as an encoder for an algebraic code -- that is, to produce the check symbols from the input message digits.

The received digits are then decoded by an ordinary algebraic decoder. Computer simulations indicate that under certain conditions, orders of magnitude improvement on error rate can be achieved by this method over the usual methods of transmission. (See III-10.)

Three studies were conducted of algebraic sequences with certain desirable properties. First, several sets of sequences were synthesized for which every pair of the set has a zero cross-correlation for all time shifts. These sequences were applied to the problem of designing carriers for an asynchronous multiplex system. (See IV-2. and IV-3.)

An investigation of the properties of subsequences of long pseudo-random sequences was completed. These properties in the form of moments of the weight distribution are shown to provide a method for selecting good pseudo-random sequences for correlation-detection applications. (See II-6.)

Also, the synthesis of two-dimensional [illegible]s exhibiting good two-dimensional autocorrelation properties and cross-correlation was accomplished. Special arrays were constructed that had two-level autocorrelation functions. These arrays may have applications in optical signal processing and pattern recognition. (See II-1 and IV-9.)

Stochastic Systems

Two problems were considered in the area of identification of unknown systems or processes. An optimum sampling rate was found for modeling a continuous time system by a discrete time system model. It was shown that the common assumptions that faster sampling leads to better results is not true in general and that in at least one sense an optimum sampling rate exists for a given type of model. (See IV-1).

A second problem was concerned with finding an estimator of the autocorrelation function such that the average risk associated with the estimate is invariant to the true values of the autocorrelation (See III-1 and IV-5.)

Two investigations were conducted in the area of feedback communications. A sequential coding scheme was analyzed and compared with the conventional block coded scheme for both wideband and bandlimited channels. (See III-2.) Also a feedback scheme based upon the Kiefer-Wolfowitz stochastic approximation technique was analyzed. It was shown that for the wideband case this scheme performed the same as the usual scheme based upon the Robbins-Monro stochastic approximation procedure. (See II-4.)

Adaptive detection and learning machines were the subject of two related studies. A comparison of a taught, untaught and decision directed detectors was made. (See II-3.) Also an adaptive detection scheme based upon extreme value statistics was postulated and analyzed. This scheme used the theory of extremes to determine the behavior of the underlying probability density functions on their tails. From this information the best threshold was determined. In the case of a fading signal, the receiver became adaptive with a time varying threshold. (See III-4 and IV-13.)

The transmission of the linear sum of $m > 2WT$ biphase modulated signals in a time interval T was considered for an additive Gaussian white noise channel of bandwidth W. It was found that if all m signals were independently modulated the error probability can be made to go to zero exponentially with 2WT if the signal to noise ratio increases linearly with 2WT. It was also shown that if k m are independently modu-

lated the remainder carving redundant information, for a fixed signal to noise ratio the error probability can be made to go to zero for $k/2WT < C^*$. For high signal noise ratios $C^*$ is greater than 1. (See III-5, IV-6 and IV-12.)

The asymptotic probability of error for the transmission of a large number of orthogonal signals over a generalized incoherent channel was also investigated. (See III-5 and IV-11). Another investigation was concerned with bounds for the probability of error for a binary input Gaussian channel where the output is quantized to Qlevels. Three situations were considered: no feedback, decision feedback and information feedback. (See III-7.)

Stochastic sources were also investigated. The rate-distortion function for a Gaussian-Markov source was investigated for a mean-square distortion measure. It is shown that if a feedback channel is available, the ideal performance can be achieved using a simple scheme. The ideal performance is only achieved with infinite delay. The rate of convergence to the minimum mean-square error as a function of delay was found for both feedback and nonfeedback schemes. A differential PCM and an adaptive quantizing system were found to perform within 4.34db of the rate distortion function bound. (See III-8, IV-16 and IV-18.)

Finally, the subject of probabilistic decoding was considered. In particular, the performance of a multiple-access communication system was computed with probabilistic decoding used on the links. Performance curves showing tradeoffs between power and bandwidth were calculated. (See II-5.)

The objectives of university research are varied. Two of the prime objectives are the education of new research workers and the pursuit of new knowledge. It is felt that both of these objectives were achieved under this contract.

During the past five years, six master's projects and ten doctoral dissertations were completed under sponsorship (or partial sponsorship) of this contract. Other doctoral dissertations are currently in progress but were not reported here. These students, the project director and the Polytechnic Institute of Brooklyn are deeply appreciative to the Office of Scientific Research of the United States Air Force for the essential part they have played in this program.

# II. MASTER'S REPORTS

## II-1. "Correlation of Doubly Periodic Arrays" by Domenick Calabro - 1967


Abstract

This report is concerned with the correlation properties of two dimensional arrays with binary and nonbinary elements. The arrays are formed by repeating a basic block of elements on all four sides and diagonally producing a doubly periodic structure. The correlation function of these arrays now involves two dimensional shifting and addition operations. The ultimate goal is to synthesize arrays exhibiting prescribed correlation properties.

The first part of this report concerns the synthesis of binary and nonbinary (N-ary) arrays exhibiting two level autocorrelation functions. In the second part of this report arrays exhibiting perfect periodic cross-correlation functions (zero cross-correlation for all cyclic shifts) and good autocorrelation functions are constructed.

II-2. "Linear Coding Techniques for Unequal Error Protection"
by Gerald Pasternack - 1967

Abstract

Although intensive research is being conducted in the area of linear coding theory, comparatively little concerns itself with "Unequal Error Protecting" (UEP) codes. These are codes which have the capability of providing different levels of error protection for specific digits of the code word - as opposed to the more conventional "Equal Error Protecting" (EEP) codes in which the digits of the coded word are all protected to the same degree.

This paper presents several results pertinent to UEP codes. A theoretical basis for these codes and their relation to EEP codes has been developed by other authors and these results are briefly discussed. Next, an algorithmic technique for code construction is established. By using this procedure, a specific UEP code of minimum length may be constructed. The methods of integer linear programming are employed and the application of the "simplex algorithm" is discussed in detail and illustrated. Upper bounds on both the computational complexity of the simplex algorithm, as well as code parameters are derived. Finally, the problem of decoding is investigated, and the theoretical basis for two iterative techniques is developed.

Each of the concepts presented are illustrated with practical examples.

II-3. "Some Learning Machines for Pattern Recognition"
by John Robert Clark - 1968

Abstract

In this report we examine some basic pattern recognition machines with the learning feature. The discussion is restricted to parametric learning of the unknown, constant mean vector of a pattern class.

A machine is formulated in general terms, and methods for realizing various learning algorithms are indicated. Both Bayesian decision theory and stochastic approximation are used in the derivations. A specific example - a single-sample binary detector for an unknown signal in additive, white, gaussian noise - is examined in detail. Performance figures are obtained whenever possible for various learning algorithms: taught, untaught, decision-directed, and combination taught-decision-directed. Each results are obtained in the performance analyses of the taught and untaught machines, but only approximate results are obtained for the other machines. However, a straightforward but tedious procedure for finding exact answers is outlined. The taught machine is found to be the best in almost all respects, followed closely by the decision-directed machine at all but the poorest signal-to-noise ratios. The untaught machine is more appealing than the decision-directed detector at low signal-to-noise ratios, where the latter becomes strongly biased in the estimate of the decision threshold.

Results of computer simulations are presented; these show good agreement with the theoretical predictions.

II-4. "A Feedback Communication Scheme Based on the Kiefer-Wolfowitz Procedure"
by Richard W. Muise - 1968

Abstract

A feedback communication scheme has been studied by Kailath and Schalkwijk which is based on the Robbins-Monro stochastic approximation technique. They assumed that the channel consisted of a noiseless feedback link and an additive white Gaussian noise forward link. The analysis was treated in two parts, first with no bandwidth constraint on the signals and second with a finite bandwidth constraint on the signals. This report discusses a feedback communication scheme subject to the same assumptions and constraints but based on the Kiefer-Wolfowitz stochastic approximation technique. It is shown that the performance achieved with this scheme is the same as that achieved with the Schalkwijk-Kailath scheme for no bandwidth constraint. However, then a finite bandwidth constraint is imposed on the signals, the Schalkwijk-Kailath scheme is shown to perform better, with the difference in performance becoming arbitrarily small as the bandwidth is allowed to increase without limit.

The two feedback schemes are then compared with the best known (simplex signals) one way scheme and are shown to have better performance than the one way scheme.

II-5. "Coding for Multiple-Access Communication Systems"
by Nicola A. Macina - 1969

Abstract

To provide a communication system with multiple-access capability imposes certain sacrifices in overall system efficiency; for a given amount of information to be exchanged over the system, more power and bandwidth are required than for more conventional systems. Of the many techniques proposed and analyzed over the past years, a few seem to benefit from the application of coding theory to their design. This report analyzes one of these techniques, a Pulse-Address Multiple Access (PAMA) system, for the purpose of showing the extent to which coding can improve the system efficiency in power and bandwidth utilization. A system model is first defined, patterned after a satellite communication system, and the performance of a non-coherent block-orthogonal signaling scheme is calculated. Next, algebraic coding is applied and shown to improve the performance significantly, particularly in bandwidth, where a reduction by almost three orders of magnitude is achieved. Finally, probabilistic coding (sequential decoding) is applied, and two deciding strategies analyzed: Hard decision, and detection with list. Further improvements over algebraic coding are realized by amounts generally commensurate to the mechanization complexity. For each case, performance curves are presented as tradeoffs between power and bandwidth requirements to guide the designer in the choice of the best engineering compromises. A practical example of the application of the results of the analysis concludes this report. This study is considered significant, as it presents a systematic approach to a communication system design, while proving the effectiveness of coding to improve the efficiency of a particular category of communication systems.

II-6. "Properties of Subsequences of Pseudo-Random Sequences" by S. Wainberg - 1969

Abstract

One method of synchronizing a communication system uses the crosscorrelation between a maximal length pseudorandom, binary sequence and a portion of that sequence. Using only part of the sequence causes the correlation function to have peaks when the sequences are not aligned, and thus may cause A threshold decision scheme to synchronize incorrectly. The object of this report is to investigate the properties of the subsequences of long psuedorandom, binary sequences. These properties, in the form of the moments of the weight distribution of the subsequences, are shown to provide a practical aid for selecting good m-sequences for various correlation-detection design problems. In particular, the first four moments are described in detail for subsequence lengths up to one hundred for six different m-sequences. Using the moments, a relationship is shown between the subsequences of the pseudorandom binary sequence and subsequences composed of random, binary numbers. A technique is shown for finding the characteristic polynomials or generating functions which will produce an m-sequence that has subsequence properties similar to that of random number sequences. An attempt is made to determine the value of sampling the m-sequence to find the moments of the weight distribution of the subsequences. Again a comparison is made to the random number situation. These investigations enable us to model some m-sequences by random, binary, numbers.

## III. Ph.D. DISSERTATIONS

### III-1. "Invariant Estimation of Stochastic Systems Parameters" by Guner Suzek - 1966

Abstract

This dissertation is concerned with the estimation of the statistics of a class of random processes. Properties of a new constant risk estimator of the autocorrelation function for exponentially correlated processes are studied analytically. The results are applied to computer-generated random processes and the estimates found are compared with the results of more common estimation methods, such as autoregressive estimation. The effect of increasing the amount of data and changing the input distribution is examined both analytically and experimentally.

III-2. "Some Results for Additive Noise Channels with Noiseless Information Feedback"
by Thomas W. Eddy - 1968


Abstract

This thesis presents some new results for two distinct aspects of the additive noise channel with noiseless information feedback. First, the performance capabilities of channels with additive colored noise in the forward channel are examined. Next, a sequential coding scheme is proposed for the additive white noise channel and its performance is analyzed and compared to the performance of the Schalkwijk coding procedures. The coding procedures analyzed in both segments of the thesis employ the basic ideas of the Schalkwijk coding schemes.

In the last part of the thesis a sequential coding procedure using information feedback is proposed and analyzed. Performance curves for wideband and bandlimited channels are presented and the asymptotic probability of error is determined. For the wideband channel

$$P_e \sim \frac{2e^{-\frac{1}{2}\left(\sqrt{6}\, e^{\bar{T}(C-\bar{R})} - \frac{1+\nu}{2}\right)^2}}{\sqrt{2\pi}\left(\sqrt{6}\, e^{\bar{T}(C-\bar{R})} - \frac{1+\nu}{2}\right)}$$

where $\bar{T}$ is the average time per message, $\bar{R} = \ln M/\bar{T}$, C is the channel capacity and $\nu = 0.5772156649...$ is Euler's constant. Similarly, for the bandlimited channel

$$P_e \sim \frac{2e^{\frac{1}{2}\left(\sqrt{6}\, e^{\bar{T}(C-\bar{R})} - \frac{e}{2} - \frac{C}{W}\right)^2}}{\sqrt{2\pi}\left(\sqrt{6}\, e^{\bar{T}(C-\bar{R})} - \frac{e}{2} - \frac{C}{W}\right)}$$

Corresponding results for the Schalkwijk coding procedures are

$$P_e \sim \frac{2e^{-\frac{1}{2}\left(\sqrt{3}\, e^{R(C-R)} - \frac{1+\nu}{2}\right)^2}}{\sqrt{2\pi}\left(\sqrt{3}\, e^{T(C-R)} - \frac{1+\nu}{2}\right)}$$

and

$$P_e \sim \frac{2e^{-\frac{1}{2}\left(\sqrt{3}\, e^{T(C-r) - \frac{e}{2}^{-\frac{C}{W}}}\right)^2}}{\sqrt{2\pi}\left(\sqrt{3}\, e^{T(c-R) - \frac{e}{2}^{-\frac{C}{W}}}\right)},$$

While the error performance is significantly improved by using sequential information feedback, the sequential results are valid only if an infinite buffer is used. In the last chapter the effect of a finite buffer is examined. It is shown (for a particular algorithm) that the finite buffer introduces an additional probability of error which decreases as $1/\sqrt{B}$ where B is the maximum number of messages that can be stored in the buffer. This result illustrates that the significant role played by the buffer in the sequential coding procedure.

III-3. "Error Control on Real Channels"
by Michael Muntner - 1968

Abstract

The promise of significant improvement associated with the use of error control techniques on digital communications channels has been based upon the assumption that errors occur independently. Real channels (e.g., telephone circuits, troposcatter radio, etc.) however do not exhibit this property in that errors occur in bursts. This dissertation examines the performance of some of the more complicated coding techniques by using a model that is representative of a real channel.

This model describes the distribution of the errors in time. It relies upon the concept of a renewal channel (a channel in which the occurrence of an error removes all memory of previous errors). The model introduced in this dissertation is shown to have the following properties: ability to match the error distributions recorded on different types of channels; ease of selection of model parameters; simplification of code evaluation; stationarity; and ergodicity.

Several error control techniques were evaluated. These included recurrent codes, concatenated codes, detect and retransmit codes with delayed retransmissions and interleaved codes. In addition, the concept of space diversity coding is introduced. This technique takes advantage of the fact that while errors may occur in bursts on a given channel, bursts of errors on different channels may occur independently. This, then, encourages the user to "code across channels.

While the evaluation of codes is the primary objective of this work, some thought is given to the problem of recording the error statistics to which the model is matched. The theory of extremes is shown to require less data, when estimating the average error rate, than simply counting the number of errors in a given number of transmitted bits. Two small simulations were conducted to confirm this conclusion. Theoretical consideration is given to the estimators of the parameters in the extreme value distributions for the case of both a small and a large number of samples.

III-4. "Nonparametric Detection Using Extreme-Value Theory"
by Laurence B. Milstein - 1968

Abstract

This paper concerns itself with the detection of a binary signal in additive, but statistically unknown, noise. The signal will be either a constant signal, or a slowly fading signal. The noise will be arbitrary except for the one restriction that its probability density function exhibit some type of exponential behavior on its "tails".

The detector will be based upon Gumbel's extreme-value theory (EVT). Extreme-value theory is a branch of mathematical statistics which considers the asymptotic distributions of the maximum and minimum samples from sets of independent and identically distributed random variables. This theory will be used to obtain estimates of the optimum threshold and the probability of error of a binary detector. Confidence intervals are obtained for all estimates.

A comparison is made between the EVT detector and another nonparametric detector, one which is based upon the rank test. It is shown that in certain conditions, the EVT detector becomes identical to the Neyman-Pearson detector, and therefore will outperform the rank or any other nonparametric detector.

When the signal fades, it is shown that the EVT detector becomes adaptive and can track the fade. Computer simulations are run for a fading signal, and the results verify the theory.

Finally, while the above results are obtained with the help of an initial learning period, a study is made, for the case of detecting a constant signal in additive noise, of the performance of the detector when the learning period is removed. It is shown that for low error rates, the estimates will converge to values close to those obtained when the learning period is present. A computer simulation is run for this case, and again the results verify the theory.

III-5. "On the Transmission of Information Over the Gaussian and Related Channels"
by Leonard Schiff - 1968

Abstract

This thesis is concerned with the transmission of information from a digital source over the Gaussian channel and certain closely related channels. The vector space point of view is emphasized throughout the work so that all transmitted signals are considered equivalent to vectors in a finite dimensional space.

The first part of the thesis is concerned with binary signaling over the Gaussian channel such that every T seconds the transmitter emits the sum of m biphase modulated signals. This signaling scheme is considered with various types of receivers, with different transmitter power constraints and both with and without coding (redundancy). The transmission system performance, under the various conditions, is evaluated by bounding the error probability both from above and below. The results have application in the study of binary synchronous multiplex systems and binary transmissions (using error correction codes) at data rates higher than the Nyquist rate.

Transmission of information over the Gaussian channel at very low signal to noise ratios (i.e., the very noisy continuous channel) is also considered. Results are obtained, both with and without receiver quantization, that are analagous to results previously obtained for the very noisy amplitude-discrete channel.

The final section of this thesis is concerned with the incoherent channel (and an extension of this channel termed the "generalized incoherent" channel) for both high and low signal to noise ratios and both with and without a bandwidth constraint. For the band unlimited channel results are obtained that are similar to results already known for the use of orthogonal signals transmitted over the Gaussian channel.

III-6. "Burst-Error Correction"
by John Dewey Bridwell - 1968

Abstract

This dissertation is concerned with encoding of binary data such that errors introduced by a burst-noise channel may be corrected. Both single-burst-correcting (SBC) and multiple-burst-correcting (MBC) codes are considered. The concept of burst distance is defined and the burst-correcting properties of a code relative to its burst distance are subsequently developed.

A class of SBC codes related to Gilbert codes is analyzed. A trade-off is shown to exist between the block length and the SBC capability for a code from this class. The exact SBC ability of Gilbert codes is also derived.

Product codes are shown to have MBC capabilities. The MBC properties of product codes are found from the parameters of the subcodes. A class of product codes is defined such that single, triple and quadruple bursts as well as five single errors within a block are correctible. A simple decoding algorithm is given for this class of codes, thus indicating that product codes may be well-suited to practical applications.

III-7. "Error Bounds for the Binary Input Gaussian Noise Channel with Quantization"
by Edward A. Walvick - 1969

Abstract

The problem considered in this dissertation is the determination of some bounds on the probability of error for transmission over a discrete time, additive Gaussian noise channel.

The input to the channel is a stream of signals, each of which is a random variable, which has amplitude +A or -A. The output is a stream of random variables each of which has a Gaussian distribution with mean +A and unit variance. The output is passed through an L-level quantizer. Also a noiseless feedback link may or may not be available depending on the system analyzed.

Capacity is determined, for a given A, as a function of the number of levels of quantization. Also, for a given number of levels, optimum division of output levels is considered. Bounds on the error exponent for block coding are also found as a function of the number of quantization levels for no feedback (upper and lower bounds), information feedback (lower bound) and decision feedback (upper and lower bounds).

The results indicate that considerable deterioration in performance is obtained by making hard decisions on the output signals (two level quantization). Further, for a considerable range of A, at least half the possible improvement in performance obtainable by not quantizing the output at all, is obtained by three-level quantization. However, four level quantization is suggested as performance is then less critically dependent on the location of the thresholds which divide the quantization levels than with three level quantization.

III-8. "Rate-Distortion Functions for Correlated Gaussian Sources" by Barry J. Bunin - 1970

Abstract

The encoding of an analog signal into binary digits is a problem of major importance in modern communications. The exact specification of such signals requires in general an infinite number of binary digits. Fortunately an exact specification is rarely needed. Instead we are usually satisfied if the signal we reconstruct from the binary digits is close to our original signal. The relevant problem becomes the specification of an analog signal to within some tolerance with as few binary digits as possible.

Shannon's rate-distortion function tell us, for a given tolerance, the minimum number of binary digits that must be used.

In this dissertation we study this rate-distortion function for the case where our tolerance, or distortion, is measured by the familiar mean square error, and where the analog signal to be encoded consists of a sequence of correlated Gaussian random variables.

Shannon, in his original paper, "The Mathematical Theory of Communication," derived the rate-distortion function for an independent symbol Gaussian source. Later, Kolmogorov gave the rate-distortion function for a correlated Gaussian source, without derivation. In Chapter II, we present this result with a derivation.

In Chapter III we consider communication systems with a noiseless feedback link. It is shown that in such systems, the minimum mean square error promised by rate-distortion theory can actually be achieved, with simple, known techniques.

To achieve the minimum mean square error with feedback it is necessary to introduce, in the limit, infinite delays. By using finite delays, we fall short of achieving this minimum. In Chapter IV the rate of convergence to the minimum mean square error as a function of delay was investigated for a first order Markov source. The convergence was shown to behave as $\frac{1}{N}$ where N is the block length.

Next, in Chapter V, an upper bound on the convergence to the minimum mean square error is found, when the feedback link is removed. When a noiseless channel is used, the convergence is upper bounded by a function that behaves as $(\log N/N)^{1/2}$, and when a noisy channel is used, by $(\log N/N)^{1/3}$.

In Chapter VI, the rate-distortion function is derived for the source whose output is observed only after it is corrupted by correlated noise. The derivation is performed with block length as a parameter. This enables us to use the results of Chapter IV and V, to upper bound the convergence to the minimum mean square error, as encoding delay increases. When the noise is uncorrelated, the convergences are the same as given above.

Finally, in Chapter VII, the rate-distortion functions of the class of nth order Markov sources are studied. It is shown that these sources are equivalent to uncorrelated Gaussian sources, for bit rates greater than a certain bit rate, denoted $R_{min}$. The rate $R_{min}$ occurs in the interval (o,n) where n is the order of the Markov source. Further, the equivalent uncorrelated source can be produced by following the source with a differential predictive system.

A differential PCM system, and an adaptive quantizing system were found to perform within 4.34db of the rate-distortion function bounnd.

III-9. "A Study of Lee Metric Codes"
by Chung-Yaw Chiang - 1970

Abstract

This dissertation is mainly concerned with channels and codes for the Lee metric. The last chapter deals with codes for correcting restricted-magnitude Hamming (RMH) errors.

An introductory chapter reviews digital communication systems, block codes and literature related to this dissertation.

Several memoryless channel models for the Lee metric codes are given in Chapter 2. This chapter is also concerned with the general properties of the linear Lee metric codes. Attention is focused on the minimum Lee distance of a linear code.

Chapter 3 deals with negacyclic Lee metric random-error-correcting codes. The negacyclic Lee metric codes found by E. R. Berlekamp (1968) are modified. It results in a class of cyclic Lee metric codes. The information rate of the cyclic and negacyclic Lee metric codes is discussed.

Chapter 4 deals with cyclic and negacyclic Lee metric burst-error-correcting codes. A class of single-burst-error-correcting codes, based on Wyner's low-density-burst-correcting codes, is derived. A class of tensor product codes is shown to be multiple-burst-error-correcting codes.

Several classes of codes for the RMH error channel are derived in Chapter 5. This type of channel was considered earlier by A. D. Wyner (1966) and J. K. Wolf (1969).

III-10. "The Time Dispersive Channel as a Linear Encoder"
by Theodore J. Klein - 1970

Abstracts

The problem considered in this dissertation is that of reducing intersymbol interference caused by time dispersive channels. This is not a new problem, and much recent work has centered on the use of the Tapped Delay Line (TDL) equalizer. The conventional approach is to choose the no dispersion channel as the desired channel, and then to minimize some measure of the intersymbol interference.

The approach described in this dissertation recognizes the encoding properties of time dispersive channels. These channels process the transmitted data in much the same way as the generator of a cyclic algebraic code. Two methods of attack are taken. In the first method the code generator coefficients are used as the desired response for an otherwise conventional TDL equalizer. This method is termed the Coded Equalizer method. Transmission of k q-ary symbols through the channel in cascade with the coded equalizer results in a code word. Thus, error correction can be obtained with an ordinary algebraic decoder, and without transmission of parity symbols. In the second method the channel encoding is accepted without further processing by a TDL equalizer, and is subsequently decoded by a channel decoding matrix. This matrix is designed to minimize the additive noise variance subject to the constraint that intersymbol interference be eliminated. A modification of this method allows trading computation time for a limited amount of intersymbol interference.

Upper bounds on the probability of error are derived for both methods. These bounds are in the form of easily calculated error functions. The parameters of code block length, number of information symbols per block, and alphabet size appear explicitly.

Computer simulations confirm the derived bounds and show that under certain conditions orders of magnitude improvement in error rate can be obtained.

# IV. PAPERS

IV-1. A. I. Liff and J. K. Wolf "On the Optimum Sampling Rate for Discrete-Time Modeling of Continuous-Time Systems" IEEE Transactions on Automatic Control, Vol. AC-11, pp. 288-290, April 1966.

Abstract

The sensitivity effect in the discrete-time modeling of continuous-time systems in digital identification schemes is considered as a function of the sampling interval T. It is shown that the common assumption that the higher the sampling rate the better a discrete-time model represents a continuous-time system is not true in general. Rather, it is shown that an optimum sampling rate exists which minimizes the effect of estimation errors. Experimental results are presented which confirm the existence of this optimum sampling interval. Close confirmation is found between the theoretically predicted optimum sampling interval and the experimental results.

IV-2. K. N. Levitt and J. K. Wolf, "Cyclically Orthogonal Sequences and An Application to Asynchronous Multiplexing" presented at the 1966 IEEE International Symposium on Information Theory, Los Angeles, California, Feb. 1966.

Abstract

A set of periodic, binary sequences is said to be cyclically orthogonal if every pair of sequences has a cross-correlation function which is identically zero for all time shifts. Several sets of such sequences are presented and are applied to the problem of designing carriers for an asynchronous multiplex system. The probability of error for the system is calculated and compared to other choices of carriers.

IV-3. J. K. Wolf, "Signal Design for Multiplex Carriers" presented at the 1966 IEEE Communications Conference, Philadelphia, Pennsylvania, June 1966 (Conference Proceedings, p. 178)

Abstract

Systematic procedures for the design of spread spectrum and pulse address signals are considered. The performance of systems using various types of carriers is derived. The design techniques are based upon algebraic coding theory.

IV-4. K. Levitt and J. K. Wolf, "A Class of Non-Linear Error Correcting Codes Based Upon Interleaved Two-Level Sequences" IEEE Transactions on Information Theory, Vol. IT-13, pp. 335-336, April 1967.

Abstract

In this correspondence, we consider nonlinear binary error-correcting codes based upon the n-fold interleaving of a periodic binary sequence of length L with out of phase (normalized) autocorrelation - 1/L. Examples of binary sequences with such out of phase autocorrelation (termed two-level sequences) are: 1) maximal-length linear shift-register sequences, 2) twin-prime sequences, 3) quadratic residue sequences, and 4) Hall sequences.

IV-5. L. Shaw and G. Robinson, "Invariant Estimation of Stochastic System Parameters" presented at the IFAC Symposium on "Problems of Identification in Control Systems," Prague, Czechoslovakia, June 1967.

Abstract

This paper deals with the estimation of the coefficients of a finite-order autoregression equation. A class of estimates are investigated based upon the conditional risk, i.e., the expected loss given the true parameter values. Both an estimator and a cost function are found such that the conditional risk is independent of the true parameter values. It is not known whether the resulting estimates yield the minimum conditional risk attainable.

IV-6. L. Schiff, "Synchronous Binary Multiplex Systems" 1967 International Information Theory Conference, San Remo, Italy, September 1967.

Abstract

A synchronous binary multiplex communication system is considered whereby every T seconds the transmitter emits the linear sum of m biphase modulated sub carriers $s_i(t)$. The power in each $s_i(t)$ is constrained to be less than some fixed constant. The channel is assumed to have a bandwidth $W = \frac{n}{2T}$ and to add Gaussian white noise to the transmitted signal.

The performance of two receiver structures are analyzed in detail. The first receiver structure independently demodulates the m binary messages using m matched filters. If $\gamma \equiv \frac{m}{n} < 1$, orthogonal sub carriers result in an error probability which goes to zero exponentially with n. For $\gamma > 1$, if the $s_i(t)$ are chosen as a mapping of the $m = 2^k$ code words of the binary group code, the error probability remains non-zero even when $n \to \infty$. However, this error probability is shown to be smaller than the average error probability produced by a random choice of the $s_i(t)$.

A second receiver, that yields a minimum probability of error on the joint decision of the m binary digits is analyzed. By a random coding bound it is shown that for any $\gamma$, a set of $s_i(t)$ exist such that the probability of error approaches zero, exponentially with n. The reliability is calculated and is shown to be close to the reliability for the orthogonal case. For $1 < \gamma < 2$, a semi-orthogonal choice for the carriers $s_i(t)$ is shown to have a reliability equal to that for the orthogonal case for low signal-to-noise ratio.

IV-7. J. K. Wolf, "Decoding of Base-Choudhuri Codes and Prony's Method of Curve Fitting," IEEE Transactions on Information Theory, Vol. IT-13, No. 4, p. 608, October, 1967.

Abstract

The literature is filled with examples of a common set of equations which arise in two or more diverse applications. The purpose of this correspondence is to point out that such a situation has occurred in the two fields of 1) algebraic coding theory, and 2) curve fitting.

IV-8. B. J. Masnick and J. K. Wolf, "On Linear Unequal Error Protection Codes", IEEE Transactions on Information Theory, Vol. IT-13, No. 4, October 1967.

Abstract

The class of codes discussed in this paper has the property that its error-correction capability is described in terms of correcting errors in specific digits of a code word even though other digits in the code may be decoded incorrectly. To each digit of the code words is assigned an error protection level $f_i$. Then if f errors occur in the reception of a code word, all digits which have protection $f_i \geq f$ will be decoded correctly even though the entire code word may not be decoded correctly.

Methods for synthesizing these codes are described and illustrated by examples. One method of synthesis involves combining the parity check matrices of two or more ordinary random error correcting codes to form the parity check matrix of the new code. A decoding algorithm based upon the decoding algorithms of the component codes is presented. A second method of code generation is described based upon the observation that for a linear code, the columns of the parity check matrix corresponding to the check positions must span the column space of the matrix.

Upper and lower bounds are derived for the number of check digits required for such codes and tables of numerical values of these bounds are presented.

IV-9. D. Calabro and J. K. Wolf, "On the Synthesis of Two-Dimensional Arrays with Desirable Correlation Properties" Information and Control, Vol. 11, pp. 537-560, November 1967. Also presented at 1967 International Symposium on Information Theory, San Remo, Italy, September, 1967.

Abstract

Considerable effort has been devoted in the literature to the synthesis of one-dimensional, periodic, binary and nonbinary sequences having small values for their out-of phase autocorrelation functions. This paper considers the synthesis of two-dimensional, periodic, binary and nonbinary sequences (arrays) which exhibit similar properties for their two-dimensional autocorrelation functions. These arrays may have future application in the areas of optical signal processing, pattern recognition, etc.

Various procedures are presented for the synthesis of such arrays. Two perfect binary arrays and an infinite class of perfect nonbinary arrays are given. A class of binary arrays is presented which are the two-dimensional analogue of the quadratic residue sequences and are shown to have out-of-phase autocorrelation of -1 or to alternate between +1 and -3. The perfect maps of Gordon are shown to have all values of out-of-phase correlation equal to -1. Other methods of constructing arrays based upon good one-dimensional sequences are also discussed.

Synthesis procedures are given for constructing pairs of arrays such that their corss-correlation is identically zero for all shifts and in addition individually have good autocorrelation functions. Examples are given for the various synthesis procedures.

IV-10. M. Muntner and J. K. Wolf, "Predicted Performance of Error-Control Techniques Over Real Channels", IEEE Transactions on Information Theory, Vol. IT-14, pp. 640-650, September 1968.

Abstract

The clustering of errors on real channels seriously complicates the task of evaluating the performance of error-control techniques. A model is introduced that, in addition to having the ability to match error distributions, greatly simplifies the task of code evaluations. Having selected the parameters of the model by matching the statistics recorded by Townsend and Watts on the switched telephone network, the model is shown to predict the results of tests of an interleaved burst-error-correcting code. The utility of the model is demonstrated in the analysis of error-detection codes with delayed retransmissions.

IV-11. L. Schiff, "The Asymptotic Error Probability for Transmission of Orthogonal Signals Over the Generalized Incoherent Channel," IEEE Transactions on Information Theory, Vol. IT-15, pp. 48-52, January 1969.

Abstract

The probability of error for the transmission of one of M-orthogonal, equally likely, equal-energy signals over the white Gaussian noise channel is known. For large values of $T$ it is shown that this result is asymptotically equivalent to the error probability for the same signal set transmitted over a different channel, termed here the generalized incoherent channel of order $\nu$. For $\nu = 2$ this channel is identical to the phase incoherent channel. This problem, for larger values of $\nu$, also arises in a number of other applications, some of which are given.

IV-12. L. Schiff and J. K. Wolf, "High Speed Binary Data Transmission Over the Additive Band-Limited Gaussian Channel," IEEE Transactions on Information Theory, Vol. IT-15, pp. 287-295, March 1969.

Abstract

The transmission of the linear sum of $m$ biphase modulated signals in a time interval $T$ is considered for an additive Gaussian white noise channel of bandwidth $W$. Previous analysis consider the case where $m \leq n \equiv 2WT$. In this paper, the probability of error is derived for $m > n$, a situation which arises when the channel bandwidth is insufficient to support the data rate.

Two distinct problems are considered. In the first, termed uncoded transmission, all $m$ signals are independently biphase modulated. It is shown, for this case, that if the channel signal-to-noise ratio increases linearly $n$, the error probability can be made to go to zero approximately exponentially in $n$ for any value of $m/n$.

In the second problem, termed coded transmission, only $k < m$ of the signals are independently modulated. (The remaining $(m - k)$ signals carry redundant information). By using a suboptimum receiver, it is shown that for a fixed channel signal-to-noise ratio the error probability goes to 0 exponentially in $n$ if $k/n$ is less than some number $C^*$. For high signal-to-noise ratio, $C^*$ is greater than 1, a situation which could not occur if $m \leq n$.

IV-13. Laurence B. Milstein, Donald L. Schilling and Jack K. Wolf, "Robust Detection Using Extreme-Value Theory," IEEE Transactions on Information Theory, Vol. IT-15, pp. 370-395, May, 1969.

Abstract

The use of extreme-value theory (EVT) in the detection of the binary signal in additive, but statistically unknown, noise is considered. It is shown that the optimum threshold and the probability of error of the system can be accurately estimated by using EVT to obtain properties of the initial probability density functions on their "tails". Both constant signals and slowly fading signals are considered. In the case of a fading signal, the detector becomes adaptive. Detection of the constant signal, both with and without an initial learning period, is studied by computer simulation.

IV-14. J. K. Wolf, "Adding Two Information Symbols to Certain Nonbinary BCH Codes and Some Applications," presented at Symposium on Information Theory, Dubna, USSR, June 19-25, 1969.

Abstract

This paper is a compendium of results based on a simple observation: two information symbols caN be appended to certain nonbinary BCH codes without affecting the guaranteed minimum distance of these codes. We give two formulations which achieve this result; the second yields in information regarding the weights of coset leaders for the original BCH codes.

Single-error-correcting Reed-Solomon codes with the added information symbols yield perfect codes for the Hamming metric. We use these lengthened Reed-Solomon codes as building blocks for perfect single-error-correcting codes in another metric.

IV-15. J. K. Wolf, M. L. Shooman and R. R. Boorstyn, "Algebraic Coding and Digital Redundancy" IEEE Transactions on Reliability, Vol. R-18, pp. 91-107, August, 1969.

Abstract

The techniques of coding theory are used to improve the reliability of digital devices. Redundancy is added to the device by the addition of extra digits which are independently computed from the input digits. A decoding device examines the original outputs along with the redundant outputs. The decoder may correct any errors it detects, not correct but locate the defective logic gate or subsystem, or only issue a general error warning. Majority voting and parity bit checking are introduced and the computations are made for several binary addition circuits. A detailed summary of coding theory is presented. This includes a discussion of algebraic codes, binary group codes, nonbinary linear codes, and error locating codes.

IV-16. B. J. Bunin "Rate-Distortion Functions for Gaussian Markov Processes" Bell System Technical Journal, vol. 48, pp. 3059-3074, November 1969.

Abstract

The rate-distortion function with a mean square error distortion criterion is investigated for a class of Gaussian Markov sources. It is found that for rates greater than a certain minimum, the rate-distortion function is equivalent to that of an independent letter source. This minimum rate was found to be less than n bits per symbol, where n is the order of the Markov sequence. Comparisons between the rate-distortion function, and two quantizing systems are made.

IV-17. J. D. Bridwell and J. K. Wolf, "Burst Distance and Multiple-Burst Correction" Bell System Technical Journal, Vol. 49, pp. 889-909, May-June 1970.

Abstract

This paper is concerned with burst error, burst erasure and combined burst-error and burst-erasure correction. Part I introduces the concept of burst distance and subsequently develops burst-correcting properties of a code relative to its burst distance. Part II discussed product codes for multiple burst correction (MBC). The MBC properties of a product of two codes are derived from the properties of the original codes. The correction of spot errors is generalized to multiple-spot correction. Theorems are presented which strengthen the single-burst correcting (SBC) properties of some codes. A class of codes which corrects single, triple and quadruple bursts and 5 single errors is developed, and a decoding procedure is given. Finally, a code from the new class of MBC codes is compared with three other MBC codes.

IV-18. B. J. Bunin and J. K. Wolf, "Convergence to the Rate-Distortion Function for Gaussian Sources" Accepted for publication in the IEEE Transactions on Information Theory, 1971.

Abstract

In this paper we derive an expression for the minimum mean-square error achievable in encoding t samples of a stationary correlated Gaussian source. It is assumed that the source output is not known exactly, but is corrupted by correlated Gaussian noise. The expression is obtained in terms of the coveriance matrices of the source and noise sequences. It is shown that $t \to \infty$, the result agrees with a known asymptotic result, which is expressed in terms of the power spectra of the source and noise.

The rate of convergence ot the asymptotic results as a function of coding delay is investigated for the case where the source is first-order Markov and the noise is uncorrelated. With D the asymptotic minimum mean-square error, and $D_t$ the minimum mean square error achievable in transmitting t samples we find $|D_t - D| \leq O\left((t^{-1}\log t)^{1/2}\right)$ when we transmit the noisy source vectors over a noiseless channel, and $|D_t - D| \leq O\left((t^{-1}\log t)^{1/3}\right)$ when the channel is noisy.

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# SYSTEMS THEORY APPLICABLE TO AIR FORCE COMMUNICATION AND INFORMATION HANDLING PROBLEMS

Scientific Final

Jack K. Wolf

September 1970

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This report summarizes the results of five years of research in the general areas of algebraic systems and stochastic systems. Topics considered related to algebraic systems include: unequal error protection codes, burst and multiple burst correcting codes, codes designed for the Lee metric, improved BCH codes, decoding of BCH codes, system reliability through algebraic coding, channel modeling of real channels, the use of a time dispersive channel as an encoder and the study of certain algebraic sequences. In the study of stochastic systems the following topics were considered: optimum sampling, invariant estimators, feedback communications, adaptive detection and learning, high speed binary data transmission over a-band-limited channel, multiple access communications and rate distortion functions for stochastic sources.

Only a brief summary of the research results is presented. However, abstracts of papers and dissertations resulting from this research are included.

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